PHYS 771 – Relativity and Gravitation

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Course Outline

This course introduces the mathematical methods (based around various concepts in differential geometry) used in General Relativity, which will make up the first half of the course. In the second half of the course we will apply these methods to examples found in astrophysics, including:

- Gravitational Waves
- Black Holes (and other compact astrophysical sources)
- Cosmology

As the study of General Relativity covers an incredibly wide spectrum of topics, this course is intended primarily as an introduction to the broader concepts of our current preferred theory of Gravitation, thus allowing students to identify topics for more in-depth studies following the completion of this course.

Lecture plan

1. **Introduction** – Geometric viewpoint on physics.
2. **More on Geometry** – Vectors and tensors; derivatives of tensors; number flux and conservation laws.
3. **More on tensors** – Contractor of tensors; the dual nature of vectors and their associated 1-form.
4. **Geometry and physics** – Volumes and volume elements; how to go between differential and integral formulations of conservation laws. Electrodynamics in geometric language. Introduction to the stress-energy tensor.

5. **More geometry and physics** – Physical meaning of the stress-energy tensor. Prelude to curvature: special relativity and tensor analyses in curvilinear coordinates. Derivatives and the Christoffel symbol.
6. **The principle of equivalence** – The use of freely-falling frames as a generalization of inertial frames; different forms of the equivalence principle.
7. **The principle of equivalence, continued** – Using coordinate transformations to make the spacetime "as flat as possible" in the neighborhood of an event; what we learn from this exercise. Derivatives on curved manifolds, and notions of transport. Parallel transport.
8. **More transport** – Lie transport, the Lie derivative. Symmetries of spacetime and Killing's equation. Tensor densities; applications of the metric determinant.
9. **Kinematics of point bodies** – Geodesic trajectories; symmetries of spacetime and conserved quantities.
10. **Curvature tensors** – The Riemann curvature. (Lecture also covers the Newtonian limit.)
11. **More curvature** – The Ricci curvature; the equation of geodesic deviation; the Bianchi identity.
12. **An equation for gravity** – The Einstein curvature and the Einstein field equation.
13. **An equation for gravity, path 2** – The Einstein field equation via a variational principle.
14. **The linearized limit 1** – How to sensibly define the linearized limit of the Einstein field equation. Gauge freedom; the Newtonian limit revisited.
15. **The linearized limit 2** – Characterization of the gauge-independent degrees of freedom in linearized theory. A wave equation for gravity.
16. **Gravitational radiation 1** – Characteristics of solutions to the wave equation for gravity. Action of these waves on matter; the quadrupole formula for the wave amplitude.
17. **Gravitational radiation 2** – Energy carried by gravitational waves; the quadrupole formula for wave power.
18. **Cosmology 1** – Solving the field equations by assuming an underlying symmetry. Maximally symmetric spaces and spacetimes; Robertson-Walker metrics; the Friedmann equations; matter and sources on cosmological scales.
19. **Cosmology 2** – How to connect models of the universe to things we can observe: redshift, distance measures. Sketch cosmic inflation and the problems it solves.
20. **Compact sources 1** – A spherical symmetric body that occupies a finite region of space. Birkhoff's theorem; interior and exterior metrics; the Schwarzschild solution. The Tolman-Oppenheimer-Volkov (TOV) equations of stellar structure.
21. **Compact sources 1** – Properties of solutions of the TOV equations; equations of state, the existence of a maximum mass for fluid bodies. Beginnings of black holes.

22. **Black holes** – The nature of spacetimes that are Schwarzschild everywhere; event horizons, coordinate versus physical singularities. A brief discussion of other black hole solutions and the "no hair" theorem.
23. **Orbits of black holes** – Equations governing the motion of a body orbiting a black hole. Conserved quantities; innermost stable orbits; highly non-Newtonian properties. Bending of light and the light ring.
24. **The Kerr spacetime 1** – The Kerr metric; Multiple singularities and horizons; Spins of astrophysical black holes.
25. **The Kerr spacetime 2** – Symmetries and conserved quantities; Return of the Killing vectors
26. **The Kerr spacetime 3** – Frame dragging; Orbits and geodesic motion.
27. **Beyond symmetry and linearized theory 1** – How to solve the Einstein field equations for realistic situations for which there is no exact symmetry, and the spacetime is not nearly flat. Part 1: The post-Newtonian expansion and black hole perturbation theory.
28. **Beyond symmetry and linearized theory 2** – How to solve the Einstein field equations for realistic situations for which there is no exact symmetry, and the spacetime is not nearly flat. Part 2: Reformulating the Einstein field equations for numerical analysis; numerical relativity.

Recommended Materials

- Primary reference
    - Spacetime and Geometry: An Introduction to General Relativity - 1st Edition
      Sean M. Carroll
      ISBN-13: 978-1108488396
- Additional references
    - A First Course in General Relativity - 3rd Edition
      Bernard Schutz
      ISBN-13: 978-1108492676
    - Gravitation
      Charles W. Misner, Kip S. Thorne & John A. Wheeler
      ISBN-13: 978-0691177793

There are many textbooks covering General Relativity, with their individual strengths and weaknesses. While I'll primarily be using *Carroll* during the course, students are recommended to look through other books as well, in case their presentation and descriptions of the topics in the course is more to their liking.

Learning outcomes

After completing this course, each student should:

- Understand and make use of the connections between physics and geometry as described in General Relativity.
- Apply this knowledge to astrophysical systems and be able to describe their observables.

Course Structure

Class Structure

The class will consist primarily of blackboard lectures. Students are encouraged to participate in the lectures, and questions, comments and clarifications are more than welcome!

Assessments – Problem Sets

The course will be assessed through a series of Problem sets, one per week for the first 11 weeks of the course.
Students are encouraged to collaborate on the P-set, but each student is required to write up and submit solutions in their own words.
Each P-set will contribute equal weight to the final course grade.

UNLV Policies

You can find up-to-date versions of the UNLV policies applicable to this course on this website.